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TECHNICAL TRANSLATION

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PAPERS ON ANALYSIS OF ORBITS

By Yu. V. Batrakov, et al

Translation of Bulletin of the Stations for Optical Observation of Artificial Earth Satellites, No. 7 (17), 1960. Published under the auspices of the Astronomic Council of the USSR Academy of Sciences (Moscow).

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OPTICAL OBSERVATION OF ARTIFICIAL EARTH SATELLITES (AES)

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STATIONS FOR OPTICAL OBSERVATION OF ARTIFICIAL EARTH SATELLITES

[Following are translations from bulletin issued by the Astronomical Soviet of the Academy of Sciences USSR, Byulleten' stantsiy opticheskogo nablyudeniya iskusstvennykh sputnikov Zemli (Bulletin of Stations for Optical Observation of Artificial Earth Satellites), No. 7(17), 1960, Moscow.]

PRELIMINARY ORBITAL ELEMENTS OF THE THIRD SOVIET ARTIFICIAL EARTH SATELLITE (1958 δ_2)

The table presented below gives a summary of the elements of the orbit of the third Soviet artificial earth satellite (1958 δ_2) received at the Institute of Theoretical Astronomy, Academy of Sciences USSR as

a result of the processing of visual and preliminary photographic observations made at Soviet and foreign observation stations. A brief description of the procedure used in processing the observations and calculating the orbital elements can be found in the article by Yu. V. Batrakov [1].

The following notation has been adopted for these elements:

Ω - right ascension of point of observation,
i - inclination of orbit to the equator,
ω - distance of perigee from point of observation (argument of perigee),
φ - angle of eccentricity connected with the eccentricity e in the formula $e = \sin \varphi$,
M_0 - is the mean anomaly at the instant t_0,
$\bar{n}$, n' - coefficients and expressions for the mean anomaly $M = M_0 + \bar{n}(t - t_0) + n'(t - t_0)^2$.

When calculating the semimajor axis, it is necessary to bear in mind that the average motion $\bar{n}$ includes the coefficient of the secular perturbation of the mean anomaly, caused by the oblateness of the earth M_0' and for accurate calculation of the semimajor axis by Kepler's third law it is necessary first to subtract M_0' from $\bar{n}$. This coefficient is calculated by the formula [2]:

$$M_0' = \frac{1}{2}\gamma\left(\frac{a'}{a}\right)^2 \frac{3\cos^2 i - 1}{(1 - e^2)^{3/2}} n$$

where a' is the semimajor axis of the terrestrial ellipsoid, γ is the small dimensionless value of the order of the oblateness of the earth, $n = \bar{n} - M_0'$ is the average daily movement connected with the semimajor axis in accordance with Kepler's third law

$$n^2 a^3 = fm.$$

We note, however, that when calculating M_0' it is not necessary to take the difference between $\bar{n}$ and n into consideration since the error which occurs in this case is of the second order of smallness compared with γ.

Observations were processed and orbital elements were calculated in accordance with the following values of the basic constants:

γ = 0.001625,
a' = 6378.15 km,
fm = 398590 km^3/sec^2.

TABLE

Preliminary Elements of the Orbit of the Third Soviet Artificial Earth Satellite (1958 δ_2)

No	t_0 U.T.			Ω	i	ω	φ	M_0	$\bar{n}$	n'
1	1958	October	26.0	310.82°	65.20°	358.5°	5.71°	40.59°	5009.33°	0.568°
2		November	10.0	270.78°	65.06°	353.9°	5.55°	67.79°	5026.33°	0.535°
3			21.0	241.46°	65.07°	349.2°	5.58°	339.66°	5037.64°	0.541°
4			29.0	219.46°	65.31°	345.8°	5.99°	357.18°	5047.20°	0.615°
5		December	7.0	198.15°	65.18°	342.4°	5.31°	94.54°	5056.85°	0.577°
6			15.0	176.47°	65.13°	338.9°	5.26°	264.76°	5065.70°	0.542°
7			25.0	149.14°	65.11°	335.4°	5.21°	212.86°	5075.73°	0.465°
8	1959	January	1.0	130.33°	65.27°	332.6°	5.18°	125.19°	5082.28°	0.470°
9			9.0	107.78°	65.27°	327.5°	4.80°	135.78°	5089.73°	0.478°
10			24.0	66.30°	65.33°	323.7°	4.76°	268.07°	5104.40°	0.508°
11			30.0	49.95°	65.33°	321.1°	4.76°	312.81°	5110.74°	0.553°
12		February	7.0	27.84°	65.15°	318.6°	4.89°	193.45°	5119.76°	0.573°
13			15.0	5.67°	65.22°	315.0°	4.94°	148.24°	5128.99°	0.577°
14			23.0	343.14°	65.19°	311.8°	4.80°	176.47°	5137.90°	0.523°
15		March	3.0	320.55°	65.19°	308.4°	4.72°	272.39°	5146.13°	0.509°
16			10.0	300.87°	65.06°	305.4°	4.87°	319.44°	5152.94°	0.468°
17			22.0	266.97°	65.05°	300.3°	5.10°	302.09°	5164.02°	0.470°
18			26.0	255.27°	65.05°	298.5°	5.10°	86.51°	5167.92°	0.495°
19		April	8.0	218.32°	65.05°	293.9°	5.02°	34.92°	5180.71°	0.450°
20			15.0	198.29°	65.05°	291.0°	5.02°	323.48°	5187.33°	0.474°
21			18.0	189.84°	65.12°	289.7°	4.36°	50.68°	5190.19°	0.476°
22			28.0	161.21°	65.13°	285.4°	4.32°	160.97°	5200.00°	0.494°
23		May	9.0	129.61°	65.27°	279.3°	4.39°	180.34°	5210.48°	0.484°
24			15.0	112.09°	65.14°	280.7°	4.26°	136.47°	5216.31°	0.464°
25			21.0	94.59°	65.15°	277.5°	4.27°	131.29°	5221.62°	0.429°
26			29.0	71.10°	65.31°	274.0°	4.26°	171.27°	5228.00°	0.361°
27	1959	June	4.0	53.43°	65.43°	271.3°	4.21°	232.28°	5232.19°	0.351°

Table (continued)

No	t_0 U. T.		Ω	i	ω	φ	M_0	$\bar{n}$	n'
28	July	8.0	313.87°	65.08°	256.9°	3.98°	357.04°	5258.24°	0.410°
29		13.0	299.14°	65.05°	257.7°	3.98°	14.89°	5262.51°	0.438°
30		20.0	278.47°	65.12°	245.5°	3.70°	165.32°	5268.91°	0.441°
31		25.0	263.57°	65.11°	250.5°	3.86°	233.31°	5273.56°	0.410°
32	August	2.0	239.80°	65.10°	246.6°	3.88°	328.39°	5280.24°	0.432°
33		9.0	218.99°	65.16°	243.2°	3.92°	232.24°	5286.55°	0.452°
34		17.0	195.01°	65.17°	239.4°	3.94°	74.00°	5294.01°	0.485°
35		27.0	165.00°	65.22°	235.2°	4.22°	143.72°	5303.88°	0.513°
36	September	6.0	135.36°	64.89°	230.9°	4.47°	318.60°	5315.53°	0.600°
37		8.0	129.28°	64.92°	230.0°	4.20°	151.42°	5317.82°	0.577°
38		11.0	120.20°	64.95°	228.7°	4.13°	269.90°	5321.18°	0.557°
39		23.0	83.16°	65.17°	227.3°	3.54°	124.58°	5336.00°	0.678°
40		29.0	64.96°	65.16°	227.0°	3.42°	122.89°	5334.23°	0.707°
41	October	6.0	43.57°	65.15°	224.2°	3.33°	127.74°	5354.44°	0.735°
42		10.0	31.33°	65.18°	222.6°	3.33°	316.98°	5360.15°	0.724°
43		21.0	357.40°	65.12°	218.1°	2.99°	327.49°	5376.83°	0.816°
44		26.0	341.95°	65.12°	216.1°	2.89°	232.32°	5385.14°	0.833°
45	November	8.0	301.21°	65.11°	207.1°	3.05°	184.97°	5406.86°	0.819°
46		13.0	285.45°	65.07°	204.3°	2.99°	240.48°	5415.10°	0.839°
47		18.0	269.93°	65.13°	202.8°	2.83°	336.18°	5423.64°	0.872°
48		25.0	248.30°	65.22°	206.0°	2.37°	177.58°	5436.13°	0.908°
49		29.0	235.10°	65.13°	198.3°	2.77°	343.96°	5443.48°	0.929°
50	December	5.0	215.87°	65.12°	195.3°	2.63°	279.19°	5454.91°	0.959°
51		7.0	209.35°	65.13°	194.8°	2.72°	32.65°	5458.74°	0.963°
52		15.0	183.57°	65.19°	191.3°	2.50°	205.10°	5474.56°	0.994°
53		27.0	144.78°	65.11°	185.6°	2.42°	161.16°	5498.27°	1.053°
54	1960 January	4.0	118.46°	65.18°	182.1°	2.27°	297.27°	5515.81°	1.097°
55		12.0	92.41°	65.10°	178.6°	2.04°	214.49°	5533.84°	1.147°
56		24.0	52.27°	65.29°	173.3°	2.00°	187.06°	5561.99°	1.236°
57	February	11.0	351.62°	65.11°	165.1°	1.78°	285.29°	5609.85°	1.365°
58		13.0	344.68°	65.10°	164.3°	1.77°	350.50°	5615.33°	1.372°
59		23.0	310.51°	65.10°	158.1°	1.60°	124.91°	5643.36°	1.404°
60	March	20.0	219.19°	65.10°	145.9°	1.13°	1.91°	5737.07°	2.325°

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Observations were not converted from the 1950.0 epoch to the epoch of osculation since the errors caused by failing to take the precession during this time into account would be of the same order as the errors of the observations employed. The coordinates of the observation stations were calculated relative to the true equator and the equinoxes of the epoch of osculation. Thus, taking the great parallax of the 1958 δ_2 satellite into account, the elements of orientation obtained Ω, i, and ω can be regarded as being associated with the true equator and the equinox of the instant of osculation.

The accuracy of the elements received is not high; this is particularly true of such elements as the inclination, the eccentricity, and the argument of the perigee where the reliability of observation depends to a large degree on the magnitude of the arc of the orbit included in the observations. The elements $\bar{n}$ and n' have higher accuracy since processing the observations and the derivation of systems of elements were carried out as a rule at intervals on the order of 10 - 20 days. In the majority of cases the errors in $\bar{n}$ and n' do not exceed $\pm 0.1°$ and $\pm 0.01°$ respectively.

The preliminary elements which were obtained can serve as a good basis for deriving more accurate elements by photographic observations.

The following workers of the Institute participated in the work of calculating the elements of the 1958 δ_2 satellite on the BESM computer: A. S. Sochilina, N. G. Kochina, Ye. N. Makarova, Ye. N. Lemekhova, L. A. Isakovich, R. P. Yeremenko.

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MOVEMENTS OF THE CARRIER ROCKET OF THE THIRD SOVIET
EARTH SATELLITE (1958 δ_1) AND THE MAGNITUDE OF THE OBLATENESS OF THE EARTH

The carrier rocket of the third Soviet artificial earth satellite (the 1958 δ_1 satellite) went into orbit on 15 May 1958 and was observed up to 3 December 1958, when, as a result of gradual degeneration in the orbit caused by air resistance, it entered denser layers of the atmosphere and ceased to exist. The original location of the orbit relative to the earth's terminator was such that the rocket could be observed in the Southern Hemisphere; a period of invisibility existed in the Northern Hemisphere began only in the second half of June 1958; from this time the rocket was observed in an intensive manner, both in our country as well as in other countries of the Northern Hemisphere. The considerable brightness of the rocket permitted obtaining a fairly large number of accurate
F photographic observations along with visual observations. Thus, the
7 rocket carrier of the third Soviet satellite was a very convenient object
4 for deriving elements more accurate than those obtained from preliminary processing of visual observations.

In order to derive orbital observations of the 1958 δ_1 satellite, photographic observations made at 17 Soviet stations were used: Abastumani, Alma-Ata, Ashkhabad, Baku, Kazan', Kiev, Krym [Crimea], L'vov, Moscow Astronomical Council, Nikolayev, Odessa, Pulkovo, Riga, Sverdlovsk, Stalinabad, Tashken, Uzhgorod, also some visual, photographic, and cine-theodolite observations made at foreign stations: Babelsberg (East Germany), Capetown, Johannesburg (South Africa), Mitaka (Japan), and Woomera (Australia).

These observations were published in the <u>Byulleteny stantsiy opticheskogo nablyudeniya iskusstvennykh sputnikov Zemli</u> [Bulletins of Stations for Optical Observations of Artificial Earth Satellites] published by the Astronomical Council [Soviet] of the Academy of Sciences, USSR and the Institute of Theoretical Astronomy, Academy of Sciences, USSR, also in various foreign publications. The astronomical coordinates of the stations were taken from these publications and tied in with the Krasovskiy ellipsoid.

The accuracy of the observations which were used varied.

The observations of Soviet stations were published with an accuracy of 0.1' in declination and 1 second in right ascension, 0.001 second in the instant of observation. The real accuracy of time markers was apparently somewhat lower and reached 0.01 - 0.02 seconds. Cine-theodolite observations at the Australian Woomera Station gave positions of the satellite with an accuracy of approximately 20" in azimuth and in altitude. Photographic observations from the Japanese Mitaka Station were published with an accuracy of 1 minute in both coordinates. In some cases, use was also made of visual observations at the Johannesburg Station, the accuracy of which formally reached 1 minute.

Systems of approximate elements of the 1958 δ_1 satellite obtained at the Institute of Theoretical Astronomy of the Academy of Sciences, USSR as the result of preliminary processing of visual observations were used as initial elements for improvement by means of photographic observations. The basic outlines of the procedure for obtaining preliminary elements was described previously [1].

The method we used for improving the orbital elements coincided essentially with the well-known Eckert-Brouwer method. Some difference in the formulas was caused by the fact that we made use of a rotating system of rectangular coordinates with the O_x-axis directed toward the point of osculation of the orbit. Somewhat simpler formulas are obtained in this way for calculating the coordinates of the satellite. In this system the coordinates of the satellite and the observer are calculated by the formulas

$$
\begin{aligned}
x &= r\cos u & X &= R\cos\varphi'\cos(s-\Omega),\\
y &= r\sin u\cos i & Y &= R\cos\varphi'\sin(s-\Omega),\\
z &= r\sin u\sin i & Z &= R\sin\varphi'
\end{aligned}
$$

$$
\begin{aligned}
\xi &= x - X = \rho\cos(\alpha-\Omega)\cos\delta,\\
\eta &= y - Y = \rho\sin(\alpha-\Omega)\cos\delta,\\
\zeta &= z - Z = \rho\sin\delta, \qquad (1)
\end{aligned}
$$

where x, y, z, r are the geocentric coordinates and the radius-vector of the satellite; X, Y, Z. R are the geocentric coordinates and the radius-vector of the observer; ξ, η, ζ, and ρ are the topcentric coordinates and the radius vector of the satellite. φ is the geocentric latitude of the observer; s is the local sidereal time of the observer; i is the inclination of the orbit of the satellite to the earth's equatorial plane; u is the argument of the longitude.

The differential relationships between the corrections of the spherical coordinates and the rectangular topocentric coordinates of the satellite have the form

$$
\begin{aligned}
\rho\cos\delta\, d\alpha &= -\sin(\alpha-\Omega)\, d\xi + \cos(\alpha-\Omega)\, d\eta + \rho\cos\delta\, d\zeta\\
\rho\, d\delta &= -\sin\delta\cos(\alpha-\Omega)\, d\xi - \sin\delta\sin(\alpha-\Omega)\, d\eta + \cos d\zeta \qquad (2)
\end{aligned}
$$

In turn, the corrections of the topocentric coordinates are linked with the corrections of the elements by the formulas

$$
\begin{aligned}
d\xi &= -Y d\Omega - A_1 d(M_0+w) - (r\sin u + A_1)dw + B_1 d\varphi + \left(A_1 t - \frac{2}{3n}X\right)dn,\\
d\eta &= X d\Omega - z\, di + A_2 d(M_0+w) + (r\cos i\cos u - A_2)dw + B_2 d\varphi + \left(A_2 t - \frac{2}{3n}y\right)dn,\\
d\zeta &= y\, di + A_3 d(M_0+w) + (r\sin i\cos u - A_3)dw + B_3 d\varphi + \left(A_3 t - \frac{2}{3n}z\right)dn, \qquad (3)
\end{aligned}
$$

with the following notation:

$$\left.\begin{aligned}
A_1 &= -\frac{a}{\cos\varphi}\left[\sin u + \sin\varphi \sin w\right], \\
A_2 &= \frac{a\cos i}{\cos\varphi}\left[\cos u + \sin\varphi\cos w\right], \\
A_3 &= \frac{a\sin i}{\cos\varphi}\left[\cos u + \sin\varphi\cos w\right],
\end{aligned}\right\} \qquad (4)$$

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$$\left.\begin{aligned}
B_1 &= -a\left[\sin u \sin E + \cos\varphi\cos w\right], \\
B_2 &= a\cos i\left[\cos u\sin E - \cos\varphi\sin w\right], \\
B_3 &= a\sin i\left[\cos u\sin E - \cos\varphi\sin w\right],
\end{aligned}\right\} \qquad (5)$$

$$\left.\begin{aligned}
u &= v + w, \\
\operatorname{tg}\frac{v}{2} &= \sqrt{\frac{1+e}{1-e}}\;\operatorname{tg}\frac{E}{2}, \\
E - e\sin E &= M,
\end{aligned}\right\} \qquad (6)$$

ELEMENTS OF THE ORBIT OF THE CARRIER ROCKET OF THE THIRD SOVIET ARTIFICIAL EARTH SATELLITE $(1958\,\delta_1)$

No.	t_0 U. T.		Ω	i	w	φ	M_0	$\bar{\bar{n}}$	u
1	1958, June	3.0	247°82 ±0°03	65°14	39°74 ±1°16	5°84 ±0°08	261°56 ±0°42	4986°52 ±0°01	62° - 69°
2		8.0	234.69 ±0.04	65.14	41.75	5.69	14.02 ±0.03	4995.82 ±0.03	62 - 67
3		10.0	229.47 ±0.01	65.16 ±0°01	38.31 ±0.11	5.71 ±0.01	292.06 ±0.04	5000.09 ±0.01	317 - 66
4		12.0	224.19 ±0.03	65.18 ±0.02	37.45 ±0.21	5.69 ±0.01	216.11 ±0.04	5003.70 ±0.02	333 - 68
5		14.0	218.90 ±0.01	65.15	36.90 ±0.29	5.66 ±0.01	147.18 ±0.06	5007.38 ±0.01	40 - 72
6		25.0	189.56 ±0.00	65.15	32.56 ±0.03	5.56	261.95 ±0.00	5029.07 ±0.00	103 - 138
7		26.0	186.87 ±0.21	65.14 ±0.14	31.79	5.56	252.66 ±0.12	5031.41 ±0.05	110 - 140
8		29.0	178.80 ±0.02	65.17 ±0.01	31.20 ±0.08	5.41 ±0.03	237.95 ±0.06	5039.13 ±0.00	102 - 134
9		31.0	173.43 ±0.01	65.15	30.48 ±0.11	5.47	241.05 ±0.09	5044.18 ±0.01	100 - 137
10	1958, August	2.0	168.00 ±0.08	65.16	29.01	5.57	254.94 ±0.05	5049.03 ±0.03	102 - 129
11		4.0	162.57 ±0.05	65.17 ±0.02	28.32	5.50 ±0.03	278.77 ±0.06	5054.35 ±0.01	106 - 129
12		6.0	157.30 ±0.27	65.06 ±0.17	27.54	5.49 ±0.25	312.19 ±0.43	5059.05 ±0.28	122 - 130
13		9.0	149.04 ±0.02	65.15 ±0.01	27.06 ±0.05	5.23 ±0.03	20.51 ±0.06	5066.60 ±0.01	109 - 130
14		10.0	146.37 ±0.07	65.11 ±0.04	26.58 ±0.04	5.32 ±0.13	47.91 ±0.27	5069.07 ±0.01	115 - 134
15		12.0	140.80 ±0.09	65.17 ±0.06	25.45	5.46 ±0.10	111.72 ±0.20	5074.44 ±0.05	115 - 137
16		13.0	138.12 ±0.10	65.16 ±0.07	25.04	5.15 ±0.10	147.89 ±0.21	5076.90 ±0.04	115 - 135
17		15.0	132.68 ±0.08	65.16 ±0.07	24.79 ±0.16	5.21 ±0.14	225.92 ±0.30	5081.93 ±0.01	120 - 137
18		16.0	130.00 ±0.07	65.08 ±0.05	24.22 ±0.08	5.15 ±0.08	269.47 ±0.17	5084.44 ±0.09	126 - 136
19		18.0	124.25 ±0.08	65.10	25.10	4.92	2.58 ±0.56	5090.12 ±0.03	130 - 138
20		19.0	121.75 ±0.07	65.10 ±0.06	22.59	4.90 ±0.05	56.23 ±0.12	5092.95 ±0.03	126 - 135
21		22.0	113.45 ±0.09	65.10 ±0.09	21.36	5.16	227.69 ±0.62	5102.08 ±0.19	141 - 150
22	1958, September	30.0	2.24 ±0.07	65.05	7.24 ±0.88	4.09 ±0.15	220.25 ±0.31	5243.92 ±0.02	73 - 78
23	1958, October	14.0	320.94 ±0.03	65.08 ±0.01	1.6	3.76	239.69 ±0.02	5302.71 ±0.01	98 - 120
24		17.0	312.01 ±0.02	65.00	359.07	3.56	332.61 ±0.01	5318.67	125 - 128
25	1958, October	18.0	309.16 ±0.04	64.92	358.69 ±1.81	3.52 ±0.19	254.12 ±0.44	5324.48	112 - 130
26		19.0	306.05 ±0.01	64.92	359.50 ±0.44	3.62 ±0.07	180.47 ±0.16	5330.97 ±0.01	116 - 133
27		22.0	296.81 ±0.03	64.92	0.45	3.69	0.93 ±0.04	5351.55 ±0.02	112 - 130
28		24.0	290.67 ±0.02	64.82 ±0.03	353.49 ±0.30	3.53 ±0.01	285.73 ±0.07	5366.56 ±0.03	136 - 151
29	1958, November	3.0	258.55 ±0.09	64.87	355.48	2.56 ±0.10	297.51 ±0.23	5435.09 ±0.46	63 - 79
30		18.0	210.26 ±0.03	65.03 ±0.00	346.63 ±0.20	2.22 ±0.06	238.97 ±0.06	5539.54 ±0.00	103 - 140

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and where the following notation is used:

- a is the semimajor axis of the orbit of the satellite;
- e is the eccentricity;
- w is the argument of the perigee;
- M_0 is the mean anomaly in the epoch t_0;
- Ω is the longitude of the point;
- φ is arcsin e, the angle of eccentricity;
- n is the average movement of the satellite connected with the semimajor axis by Kepler's third law.

When improving the orbital elements by these formulas, the perturbations caused by the nonspherical shape of the earth were taken into account in formulas [2] and in order to take into account the perturba-
F tions caused by air resistance, the nonlinear term $n't^2$, whose n' coeffi-
7 cient was not determined but taken from the results of preliminary pro-
4 cessing of visual observations, was added to the mean anomaly.

The possibility of isolating any of the six unknowns was provided for in calculating the elements on the BESM machine. This procedure permitted the use of the same program for different combinations of observations. The program contained provisions for converting the observed α and δ coordinates from the 1950.0 epoch to the epoch of the middle of the interval included by the observations involved in processing by introducing the corresponding corrections in precession and nutation. The observations which had been reduced to the true equator and equinox were then used for calculation of the elements. In cases in which the observations were given by the azimuth and altitude of the satellites, these figures were first converted to α and δ associated with the true equator and equinox, then were processed. Thus, the elements obtained are associated with the true equator and equinox of the middle of the interval included by the observations used in improving the given system of elements. The length of this interval varied within limits of 1 - 2 days. Therefore, the errors caused by neglecting the movement of the point of the vernal equinox and the equator in this period can be considered negligibly small.

The elements which were obtained are given in the table. In addition to giving the elements, the table also presents the mean errors if these elements were determined from observations by the least squares method. If a given element is not determined, but considered known on the basis of certain considerations, then the error of this element is not indicated. This table gives values of the argument of the latitude corresponding to the boundaries of the arc of the orbit on which the observations involved in improvement are situated. The accuracy of the elements varies from system to system; in this case, as a rule, systems defined on the major arc of the orbit.

We note that the table gives values of elements freed from the influence of first-order periodic perturbations relative to the oblateness of the earth. The mean movement contains the coefficient of the secular perturbation of the mean anomaly and cannot be used directly for calculating the semimajor axis. The semimajor axis should be calculated by the formula:

$$a = (\bar{n} - M_0')^{-\frac{2}{3}} (fm)^{\frac{1}{3}}, \tag{7}$$

The elements of the 1958 δ_1 satellite which were obtained are not final since we had the objective of obtaining a representation of the observations with accuracy on the order of only 1' - 3' after processing the data, in view of the varied nature of the data obtained from observations. However, their accuracy is higher than the accuracy of elements obtained by visual observations.

On the assumption that the dispersion of the elements obtained is of a random nature, we employed the least squares method to calculate the coefficients of polynomials which best satisfy the entire set of elements. These polynomials define the smoothed changes in elements of the rocket in the period from 3 July through 18 November 1958 and have the form F 7 4

$$\Omega = 173^\circ.421 \pm 0^\circ.030 - (2^\circ.6964 \pm 0^\circ.0015)(t - t_0) - 0^\circ.1461.10^{-2}(t - t_0)^2 - 0^\circ.467 \,.\, 10^{-5}(t - t_0)^3,$$

$$i = 65^\circ.144 \pm 0^\circ.002 - 0^\circ.125 \,.\, 10^{-2}(t - t_0), \tag{8}$$

$$\varphi = 5^\circ.450 \pm 0^\circ.039 - 0^\circ.01507 (t - t_0) - 0.1029.10^{-3}(t - t_0)^2,$$

$$w = 30^\circ.346 \pm 0^\circ.082 - 0^\circ.3868 (t - t_0) - 0.168 \,.\, 10^{-3}(t - t_0)^2,$$

where t_0 corresponds to the time 1958, July, 31.0 U. T.

The mean errors of some coefficients were usually computed by the least squares method according to deviations of elements from the smoothed polynomial values. The $\bar{n}$ element was not smoothed since its instantaneous values given in the table have negligible mean errors and, with certain exceptions, can be used without smoothing.

The parameter $\mathcal{Y}$ in the gravitational field of the earth was calculated with smoothed elements. At the same time, the analytical expression for the coefficient of movement of the point under the influence of the nonsphericity of the earth was taken in the form [2]:

$$\Omega' = \mathcal{Y}\left(\frac{a'}{a}\right)^2 \frac{\cos i}{(1-e^2)^2}\, n + \mathcal{Y}^2\left(\frac{a'}{a}\right)^4 \cos i\left(\frac{19}{6}\sin^2 i - \frac{5}{2}\right) + D\left(\frac{a'}{a}\right)^4 \cos\left(\frac{3}{2}\sin i - \frac{6}{7}\right) n \tag{9}$$

where $\mathcal{Y}$ and $\mathfrak{D}$ are the parameters of the earth's gravitational field introduced by Jeffries and a' is the equatorial radius of the earth.

In this formula it is necessary to take the difference between the tabulated $\bar{n}$ and the coefficient M_0, for n, that is, the mean movement connected with the semimajor axis by Kepler's third law.

We have the following values of the initial quantities

$$\begin{aligned} t_0 &= 1958,\ \text{July},\ 31.0 \\ i &= 65.144° \pm 0.002° \\ \Omega' &= -\ 2.6964° \pm 0.0015° \\ \varphi &= 5.450° \pm 0.039° \\ \bar{n} &= 5044.18° \pm 0.01° \end{aligned} \tag{10}$$

We also take the following values for the basic constants:

$$\begin{aligned} fm &= 398{,}600\ \text{km}^3/\text{sec}^2 \\ a' &= 6378.15\ \text{km.} \end{aligned} \tag{11}$$

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then, setting $\gamma = 0.001637$, we shall have

$$\begin{aligned} M_0' &= -\ 1.51° \\ n &= \bar{n} - M_0' = 5045.69° \\ a/a' &= 1.13928. \end{aligned} \tag{12}$$

Upon substituting these data into (9), we obtain the equation

$$-\ 2.6964° = 1662.8°\ \gamma + 137.20°\ \gamma^2 + 483.73°.$$

If we set $\gamma = 0.001637$ and $\mathcal{D} = 10.6 \times 10^{-6}$ in the second-order terms in this equation, we finally obtain

$$\gamma = (1.6249 \pm 0.0009) \times 10^{-3}, \tag{13}$$

where the mean error is evaluated by the errors of the elements which have been used. This value of the parameter corresponds to the following value of the inverse of the oblateness of the earth

$$\frac{1}{\varepsilon} = 298.17 \pm 0.08. \tag{14}$$

When $\mathcal{D}$ is decreased to 7.6×10^{-6} we obtain

$$\gamma = (1.6240 \pm 0.0009) \times 10^{-3}, \quad 1/\varepsilon = 298.25 \pm 0.08$$

These values of the inverse of the oblateness of the earth are in good agreement with the values obtained by other authors [3, 4].

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Institute of Theoretical Astronomy
Academy of Sciences USSR
Submitted to Editorial Staff
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Yu. V. Batrakov
A. S. Sochilina

ON ELEMENTS OF THE ORBIT OF THE CARRIER ROCKET OF THE THIRD SOVIET ARTIFICIAL EARTH SATELLITE (1958 δ_1) DERIVED FROM PHOTOGRAPHIC OBSERVATIONS

An article by Yu. V. Batrakov and A. S. Sochilina (Institute of Theoretical Astronomy, Academy of Sciences USSR) with the title "Movements of the Carrier Rocket of the Third Soviet Earth Satellite (1958 δ_1) and the Magnitude of the Oblateness of the Earth" was published in this issue of the Bulletin of Stations for Optical Observation of Artificial Earth Satellites. The preliminary results of the derivation of elements of the orbit of the 1958 δ_1 object are presented in the article.

Due to the inhomogeneity of the material which was used, the authors were given a restricted problem--that of obtaining a representation of the observations with accuracy on the order of only 1' - 3' and did not regard the elements they derived as final. It is not without interest, however, to compare these preliminary elements of the orbit of the 1958 δ_1 object with the results obtained by other authors.

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We have made such a comparison of the elements derived by Yu. V. Batrakov and A. S. Sochilina with the elements obtained by D. King-Hele (Notices of the Royal Aircraft Establishment, 5 June 1959).

(The King-Hele elements were given in the article by I. D. Zhongolovich in the Bulletin of Stations for Optical Observation of Artificial Earth Satellites, No. 2 (12), 1960.)

The orbital elements published by Batrakov and Sochilina are mean elements associated with the middle of the interval of time included by the observations which were used. The King-Hele elements, however. were calculated for times that the satellite passed through the ascending node. Thus, when comparing these two systems of elements defined in different ways, it is essential to take the differences in the procedure for their derivation into account, particularly the method for computing periodic perturbations.

Such comparisons are accompanied by very cumbersome computations which require the use of high-speed computers.

However, this situation is alleviated by the fact that the difference between elements caused by different determinations by different authors is of the first order of the first power of the oblateness of the earth, that is, small compared with the actual errors in the elements. This difference can be neglected in a qualitative comparison of the elements.

Four graphs (refer to Figures 1 and 2) were constructed for comparison of the orbital elements of the 1958 δ_1 object obtained by Batrakov and Sochilina with the elements obtained by King-Hele. Time is plotted along the axis of abscissas. In order to promote clarity of presentation, the quantities $\Delta\Omega$, Δi, $\Delta\omega$, and $\Delta\varphi$, which are the deviations of the corresponding elements from their values computed by interpolation formulas derived by Batrakov and Sochilina in the above-mentioned article, have been plotted on the axis of ordinates instead of the actual values of the elements. Due to the large deviations of the derived values of the right ascension of the point from their calculated values in November 1958, the scale of the axis of ordinates was decreased to one-fourth in the extreme right-most portion of the graph of $\Delta\Omega$.

An analysis of the graphs provides evidence that the values of the orbital elements computed by Batrakov and Sochilina and those of the elements derived by King-Hele agree quite well: The King-Hele elements are well represented by interpolation curves (on graphs which coincide with the axes of abscissas) computed for the elements obtained by Batrakov and Sochilina.

When making this comparison it must be borne in mind that the values of the inclinations and eccentricities were smoothed in King-Hele's determinations, but these elements were not smoothed by Batrakov and Sochilina. In individual cases in which the observed data were not sufficient for rigorous computations of all elements or when the arc of the orbit bracketed by the observations was too small, the values of some orbital elements in the work by Batrakov and Sochilina were accepted on the basis of various theoretical considerations.

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The dispersion of the points corresponding to the elements of Batrakova and Sochilina and those of the King-Hele elements are approximately the same on the graphs of $\Delta\Omega$ and $\Delta\omega$ (the elements Ω and ω derived by King-Hele were not smoothed).

Some values of the elements derived by Batrakov and Sochilina, whose derivation involved the use of observations that bracketed too small an arc of the orbit, show large deviations from the interpolation curve. Such, for example, are the values of $\Delta\omega$ associated with 3 and 8 July (corresponding to arcs of 7° and 5°) and with 24 October (an arc of 15°). (It must be remembered that as the eccentricity is decreased, that is, as the form of the orbit approaches that of a circle, the argument of the perigee w is determined less and less reliably, so that the dispersion of points on the right side of the graph is greater than on the left). An arc of 8° includes observations which were employed to derive the angle of eccentricity φ of 6 August; arcs of 8° and 5° respectively, of observations which served as material for deriving the right ascension of the point of 18 August and 30 September, et cetera.

When we were processing observations accepted on the basis of various theoretical considerations, the values of some elements did not always turn out to be fortunate. Thus, for example, the values of the argument of the perigee ω associated with 21 October and 3 November deviated considerably from the interpolation curve; this was true of the values of the angle of eccentricity φ associated with 2 and 18 August and 21 October; also the values of the inclination i associated with 18, 19, and 22 October, and a number of others.

In the next stage of the work on improving the orbital elements of the carrier rocket of the third artificial earth satellite, it will

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be possible to replace these values of the elements with those values determined from the interpolated dependence which was obtained. In this case the dispersion of points on the graphs will be decreased considerably.

I am sincerely grateful to Yu. V. Batrakov and A. S. Sochilin who kindly placed essential data at my disposal and who undertook to examine the manuscript of this article.

Astronomical Soviet of the
Academy of Sciences USSR

N. P. Yerpylev

Submitted to the Editorial Staff
22 September 1960.

A COMPARISON OF DIFFERENT SYSTEMS OF THE ORBITAL ELEMENTS OF THE THIRD SOVIET SATELLITE 1958 δ_2.

At present orbital elements of the artificial earth satellite as determined by optical observations by the Smithsonian Institute (United States), the Research Center of the Royal Air Force (England), and also by the Institute of Theoretical Astronomy, Academy of Sciences USSR have been published. A comparison of the systems of elements obtained by these institutions for the 1958 δ_2 is presented in this article.

The following data were utilized:

1. Mean elements freed from the influence of periodic perturbations. Obtained by Yu. V. Batrakov and A. S. Sochilina at the Institute of Theoretical Astronomy, Academy of Sciences USSR (published in this number of the bulletin). October 1958 - March 1960.

2. Nonosculating elements associated with the time of passage through the node. Obtained by King-Hele (D. King-Hele, Royal Aircraft Establishment, 14 September 1959, GW/1247/R T). May 1958 - May 1959. F 7 4

3. Mean elements modified for the Ephemeris Service, associated with the time of passage through the perigee. Computed and published in special card files of the Smithsonian Astrophysical Observatory. January 1959 - April 1960.

All the data from observation which were used have been published (or will be published in the future) in the bulletin Results of Observations of Soviet Artificial Earth Satellites. Satellite 1958 δ_2. On the average the accuracy of these observations is $\pm$ 1 second in time and $\pm 1°$ in position, although it is higher for some stations. Therefore, the elements were compared without taking into account the fact that their determination was different for different authors, and as a result discrepancies are possible which are on the order of the first power of the oblateness of the earth. In the case of the elements obtained by the Institute of Theoretical Astronomy, this effect is considerably less than the dispersion of elements caused by random errors.

Comparisons were made for the period P, the eccentricity e, the inclination i, the argument of the perigee ω and the right ascension of the node Ω. The results of the comparison are presented in Figures 3 - 10. The time is plotted on the x-axes of the graphs and the actual values of the elements, also the values of ΔP, $\Delta\omega$, and $\Delta\Omega$ as determined by the following formulas are plotted on the y-axes:

$$P = P_0 - [99.280 \text{ m} - 0.0222 \text{ m} (t - t_0)]$$

$$\Delta\Omega = \Omega_0 - [94.59° - 3.00° (t - t_0)]$$

$$\Delta w = w_0 - [277.5° - 0.4° (t - t_0)]$$

where P_0, Ω_0, and ω_0 are values of the elements at time t obtained from observations while t_0 is 21.0 May 1959.

The values of ΔP, $\Delta\Omega$, and $\Delta\omega$ were computed in order to be able to select a large scale when constructing the graphs, which would permit making comparisons with great accuracy. The linear terms, which are equal to 0.0222 m for the period and 3.00° for the right ascension of the node, also 0.4° for the argument of the perigee were taken directly from curves (Figures 4 - 6, 9 - 10) as mean tangents of the slopes of curves P, Ω, and ω. The values 99.280 m, 94.59°, and 277.5° are the respective values of P, Ω, and ω in the ITA [Institute of Theoretical Astronomy] system for 21.0 May 1959.

The period for ITA elements was computed by the approximate formula $P = 2\pi/\bar{n}$ and the eccentricity by the formula $e = \sin\varphi$.

The graphs (except those for i and ω) were constructed separately for 1958 - 1959 and for 1960, which also made it possible to select a somewhat larger scale.

As may be seen from the graphs, the greatest dispersion is observed in the eccentricity and inclination given by the ITA data and in the argument of the perigee by all data. This is explained by several causes.

The inclination and the eccentricity were not determined with sufficient accuracy, for example, in cases in which the arc of the orbit bracketed by the observations was small (refer to the article published in this issue of the bulletin); this is quite frequently the case for satellite observations. It is also possible that the accuracy of determination of these elements is affected by the fact that when processing observations taken over a long interval of time it is not possible to take into account short-term changes in the density of the atmosphere connected with solar activity. Moreover, it should be noted that the values of these elements as given in the data of King-Hele and the Smithsonian Institute were clearly smoothed while they were determined each time separately and not smoothed by the ITA, which explains their wide dispersion.

The argument of the perigee is not determined with sufficient reliability since the correction for this element in the conventional equations contains a factor of small magnitude--the eccentricity.

Taking the foregoing into consideration, one may conclude that there is, in general, satisfactory agreement between the ITA data and the data taken from the above mentioned sources in spite of the fact that the observations used in the ITA for obtaining the preliminary elements were of comparatively low accuracy.

The author expresses his gratitude to the ITA workers, Yu. V. Batrakov and A. S. Sochilina for advice and counsel received when preparing this article for publication.

Astronomical Council,
Academy of Sciences USSR

N. P. Slovokhotova

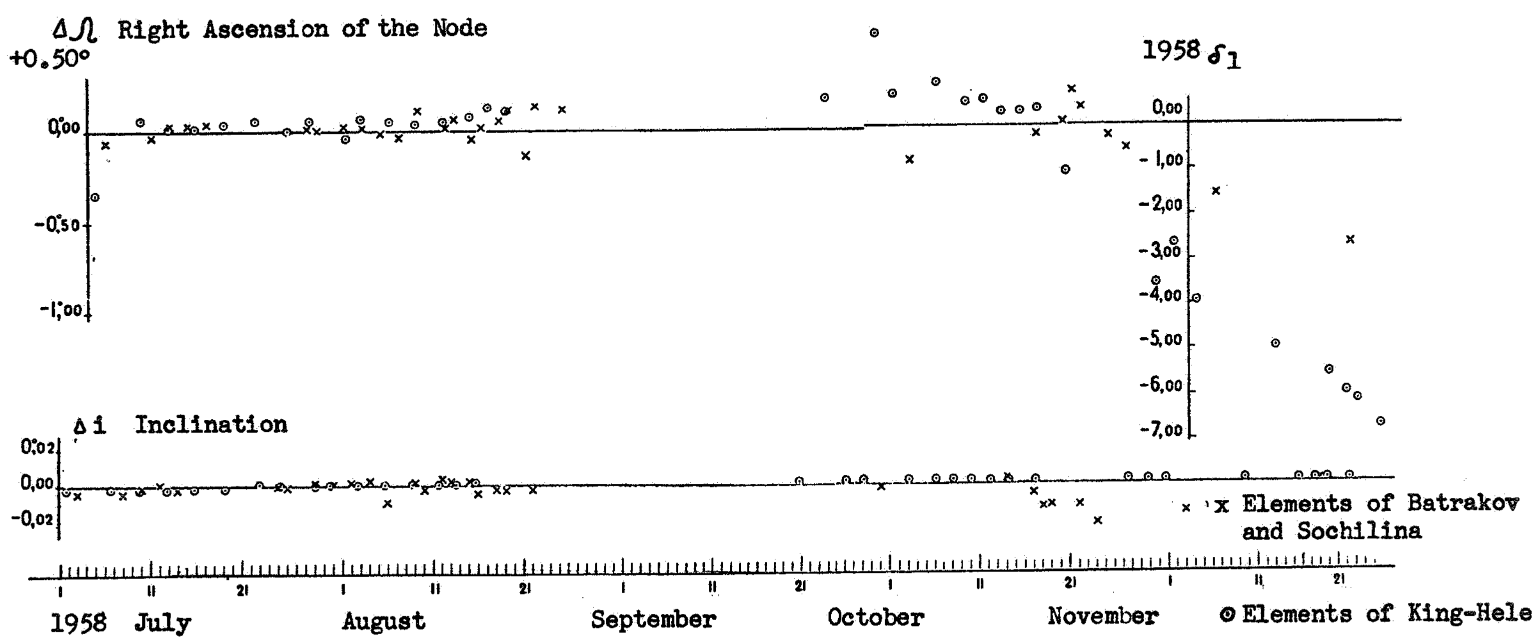

Figure 1.- (For the article by N. P. Yerpylev.)

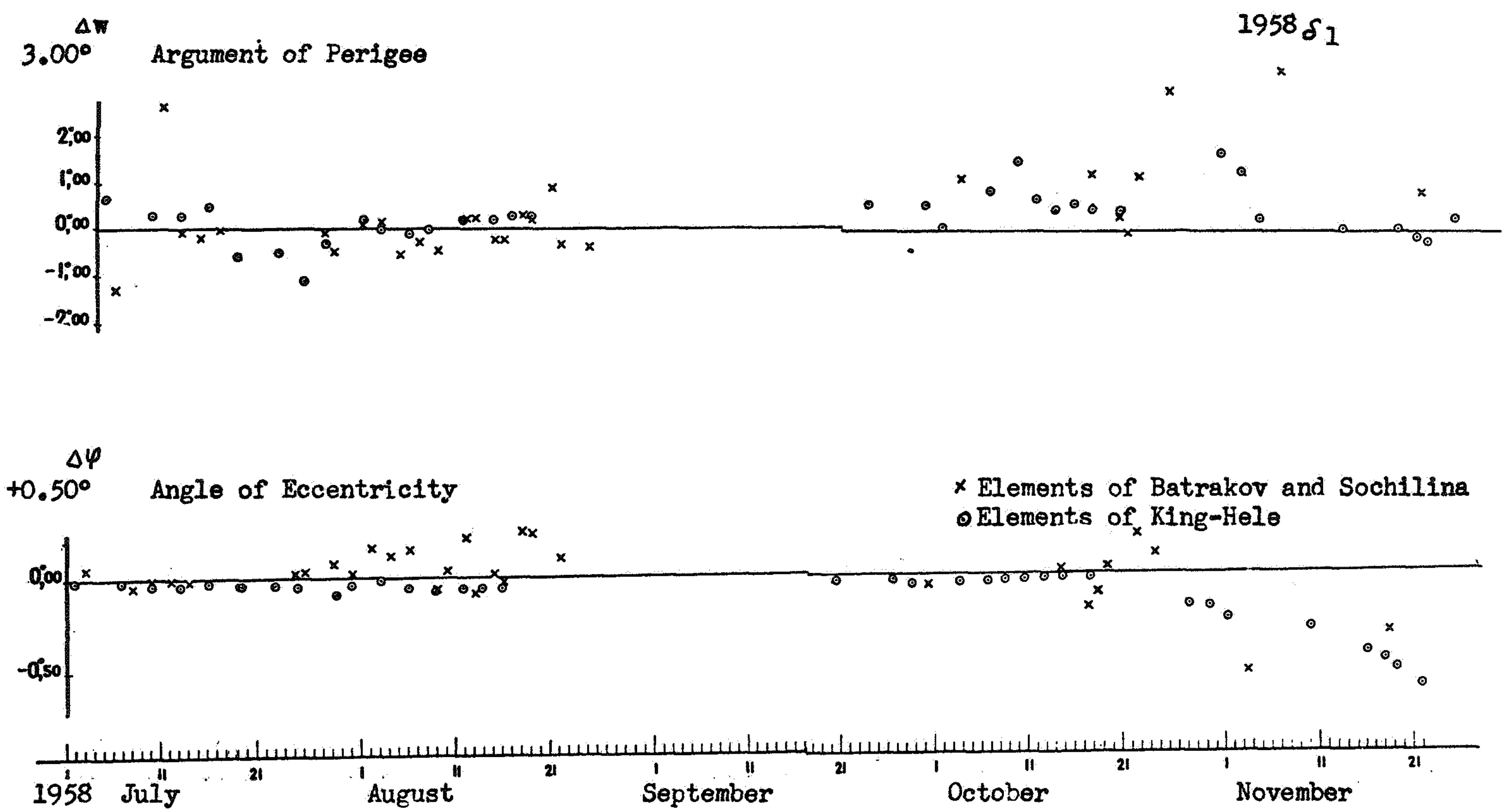

Figure 2.- (For the article by N. P. Yerpylev.)

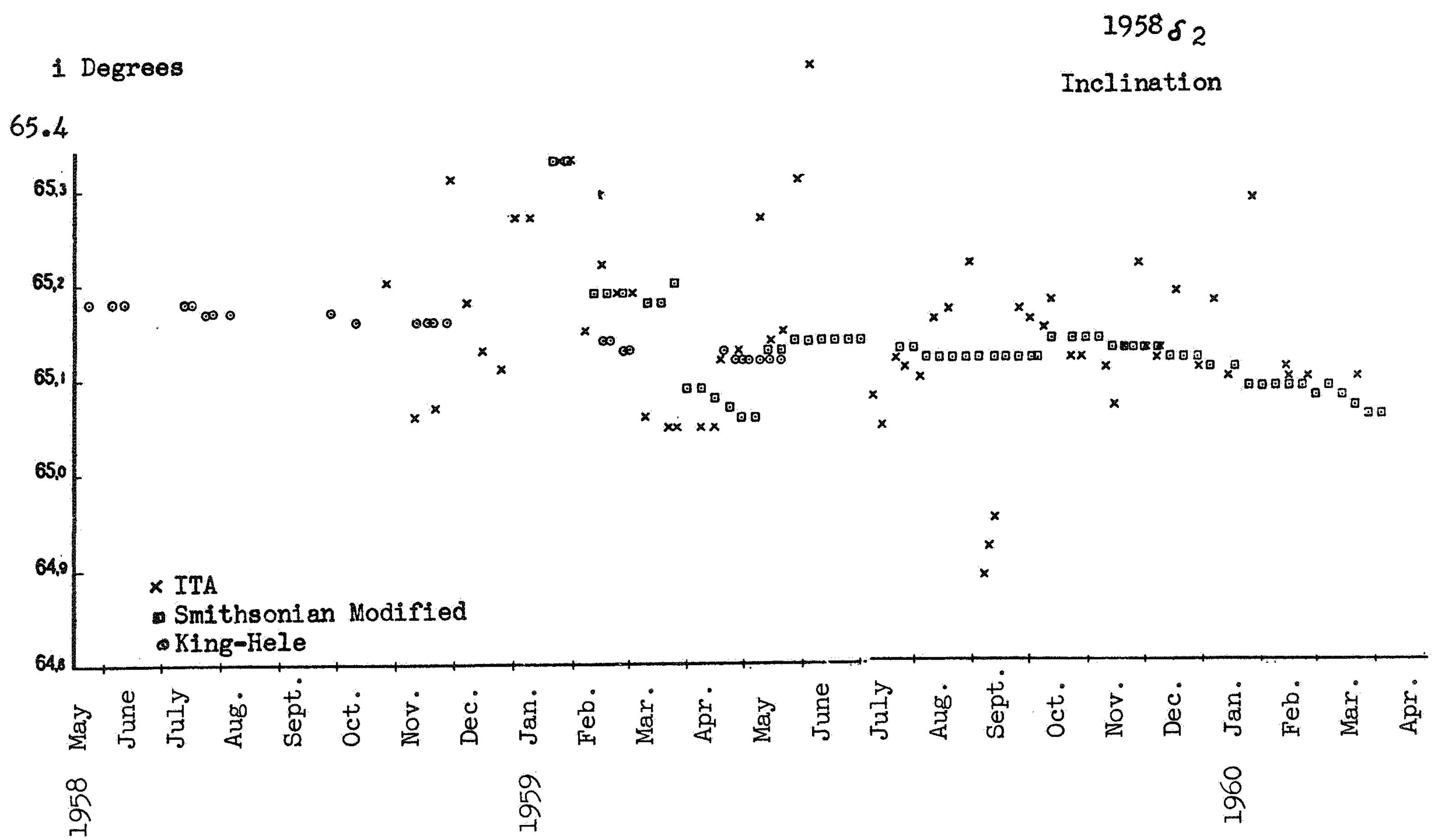

Figure 3.- (For the article by N. P. Slovokhotova.)

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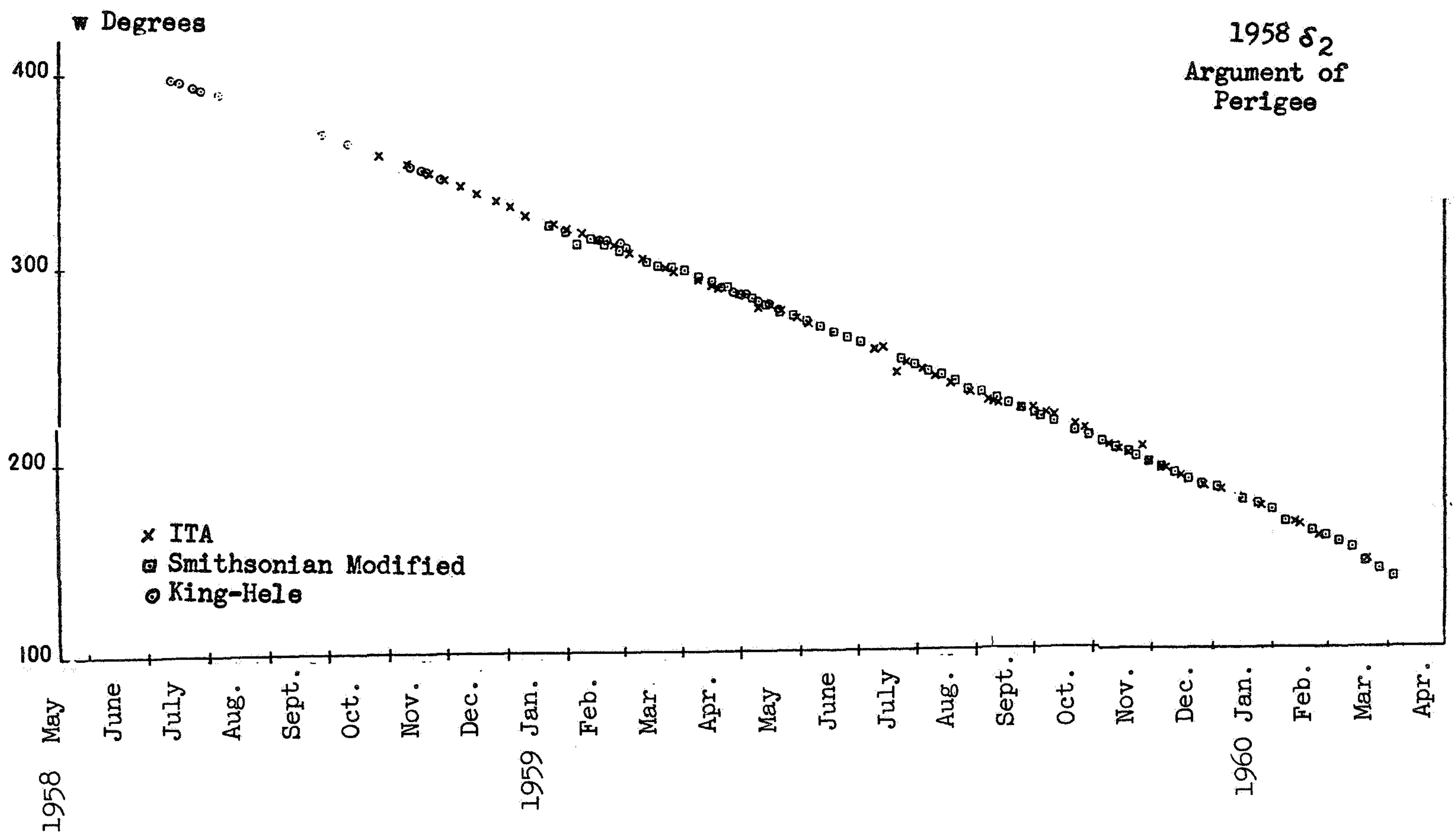

Figure 4.- (For the article by N. P. Slovokhotova.)

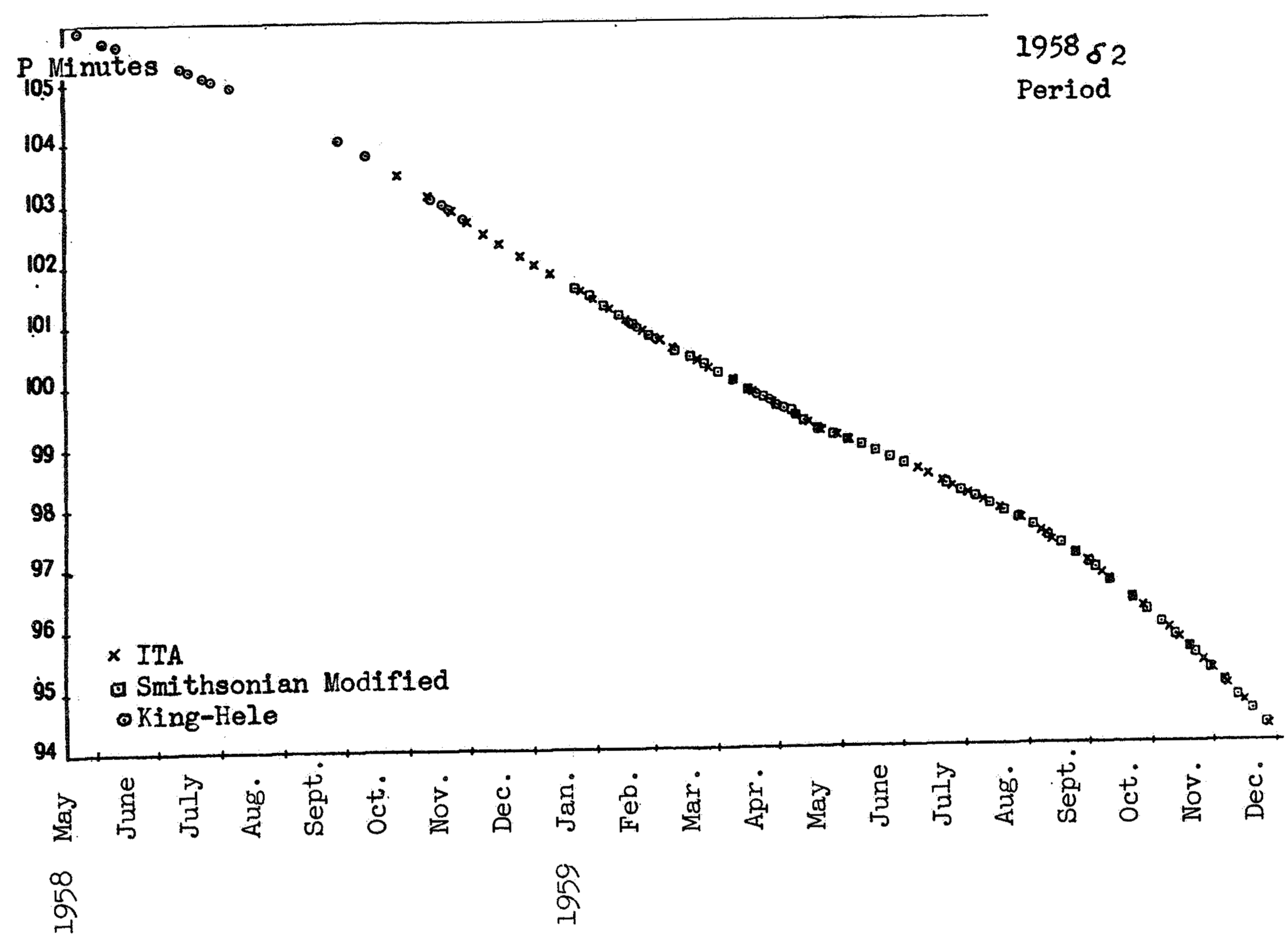

Figure 5.- (For the article by N. P. Slovokhotova.)

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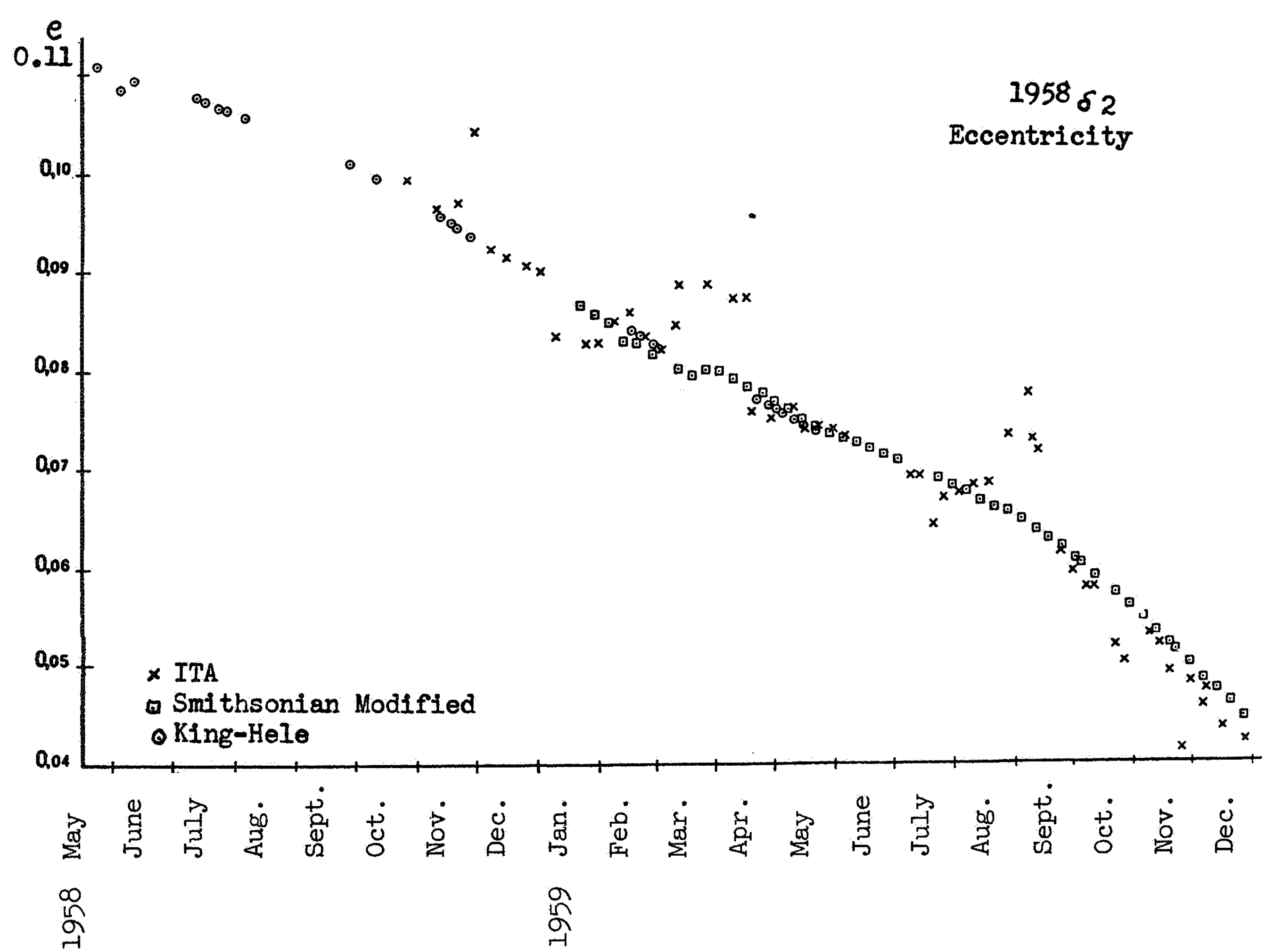

Figure 6.- (For the article by N. P. Slovokhotova.)

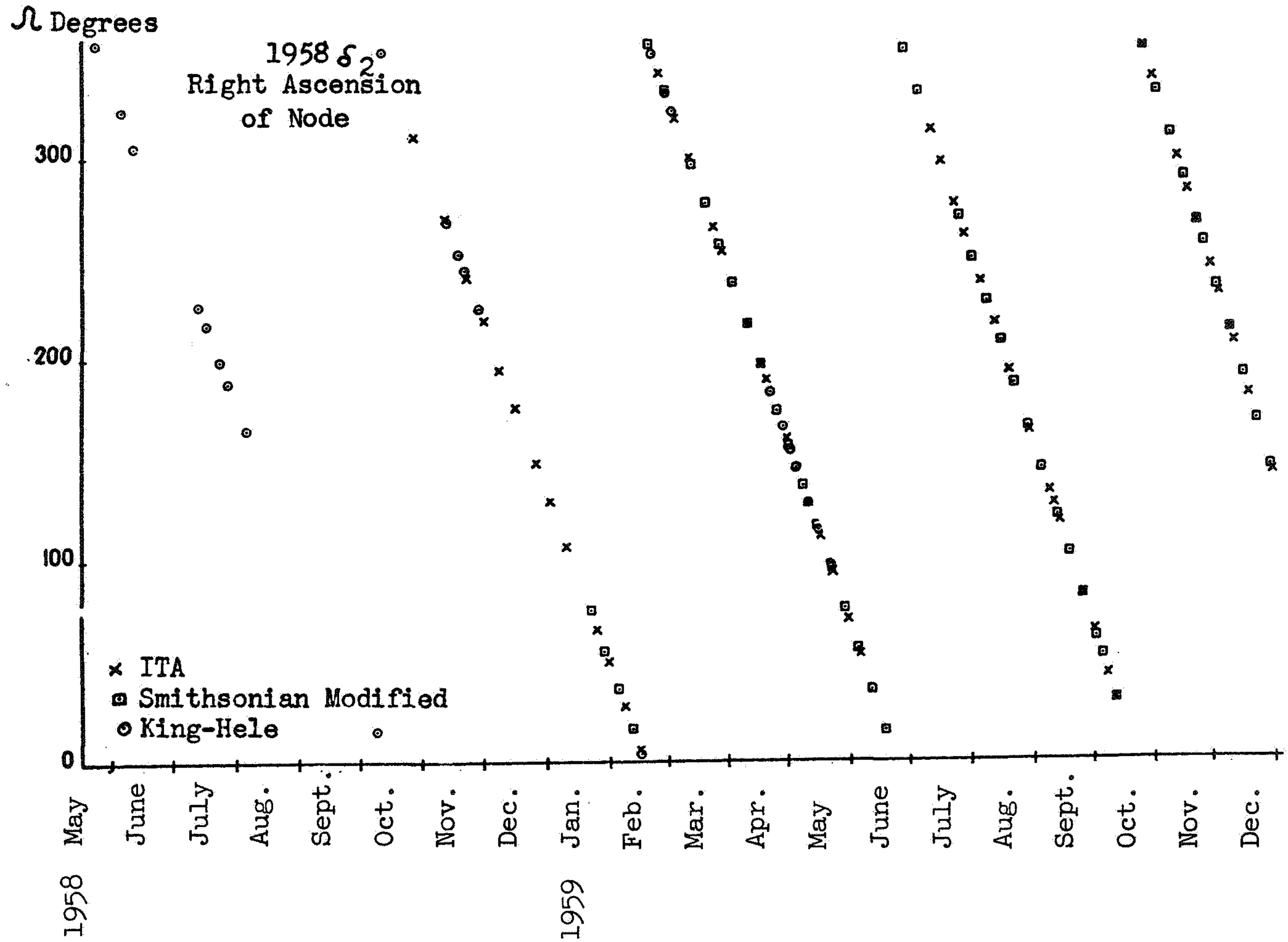

Figure 7.- (For the article by N. P. Slovokhotova.)

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1/ x - ITA
□ - Smithsonian Modified

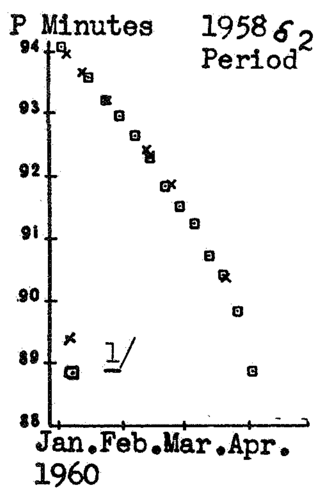

Figure 5(a).- (For the article by N. P. Slovokhotova.)

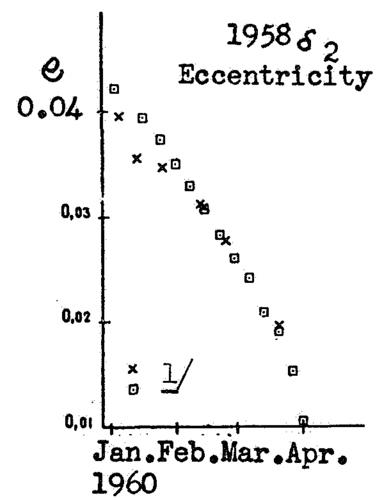

Figure 6(a).- (For the article by N. P. Slovokhotova.)

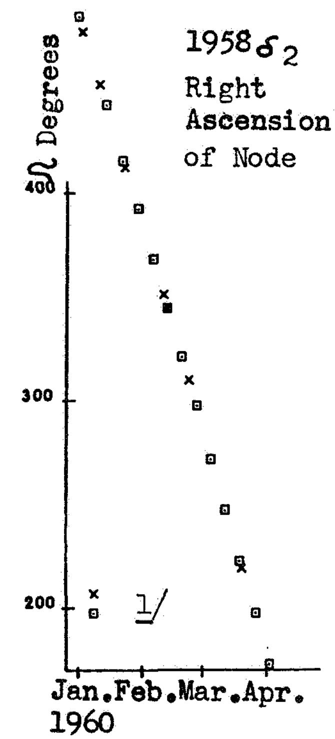

Figure 7(a).- (For the article by N. P. Slovokhotova.)

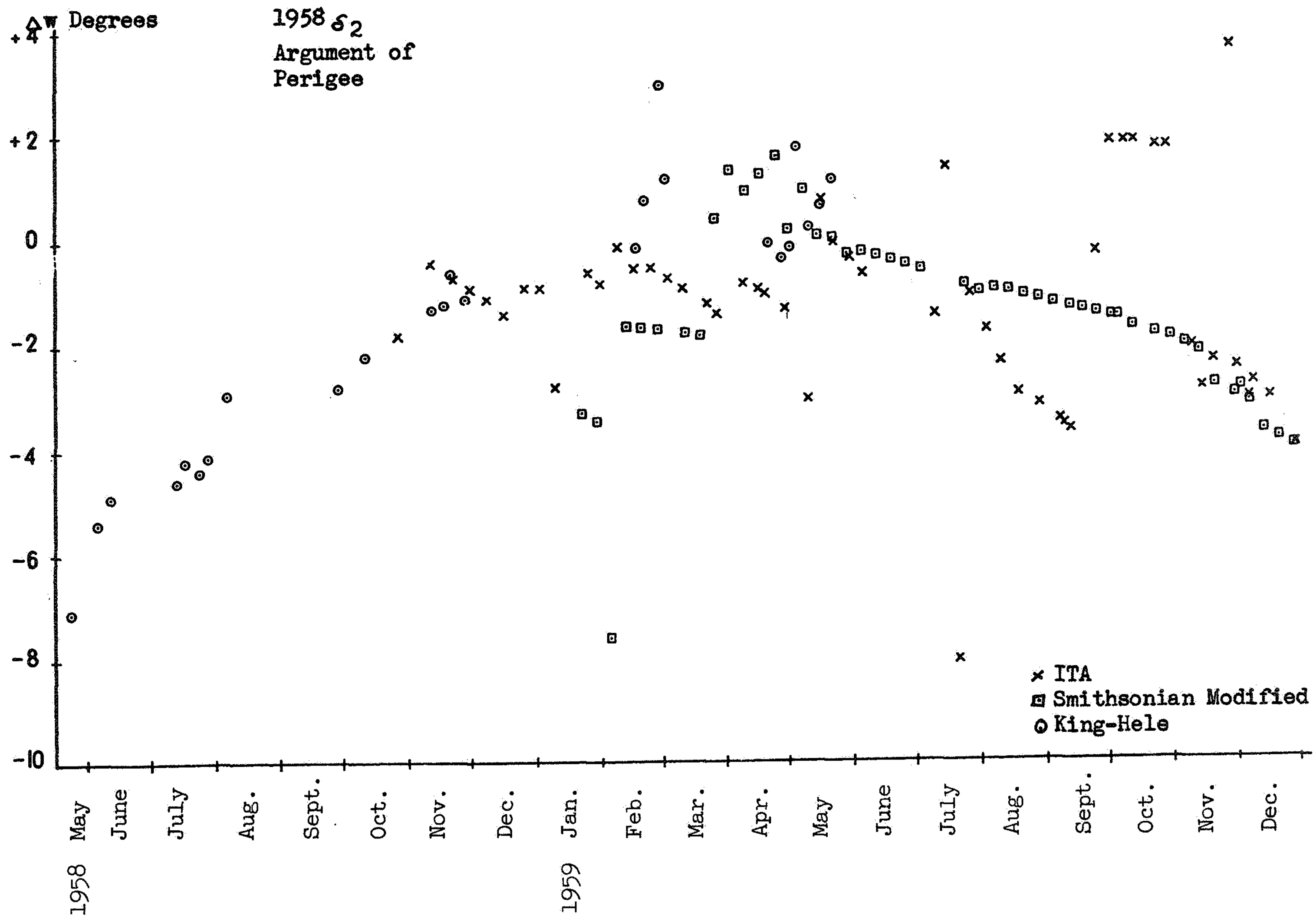

Figure 8.- (For the article by N. P. Slovokhotova.)

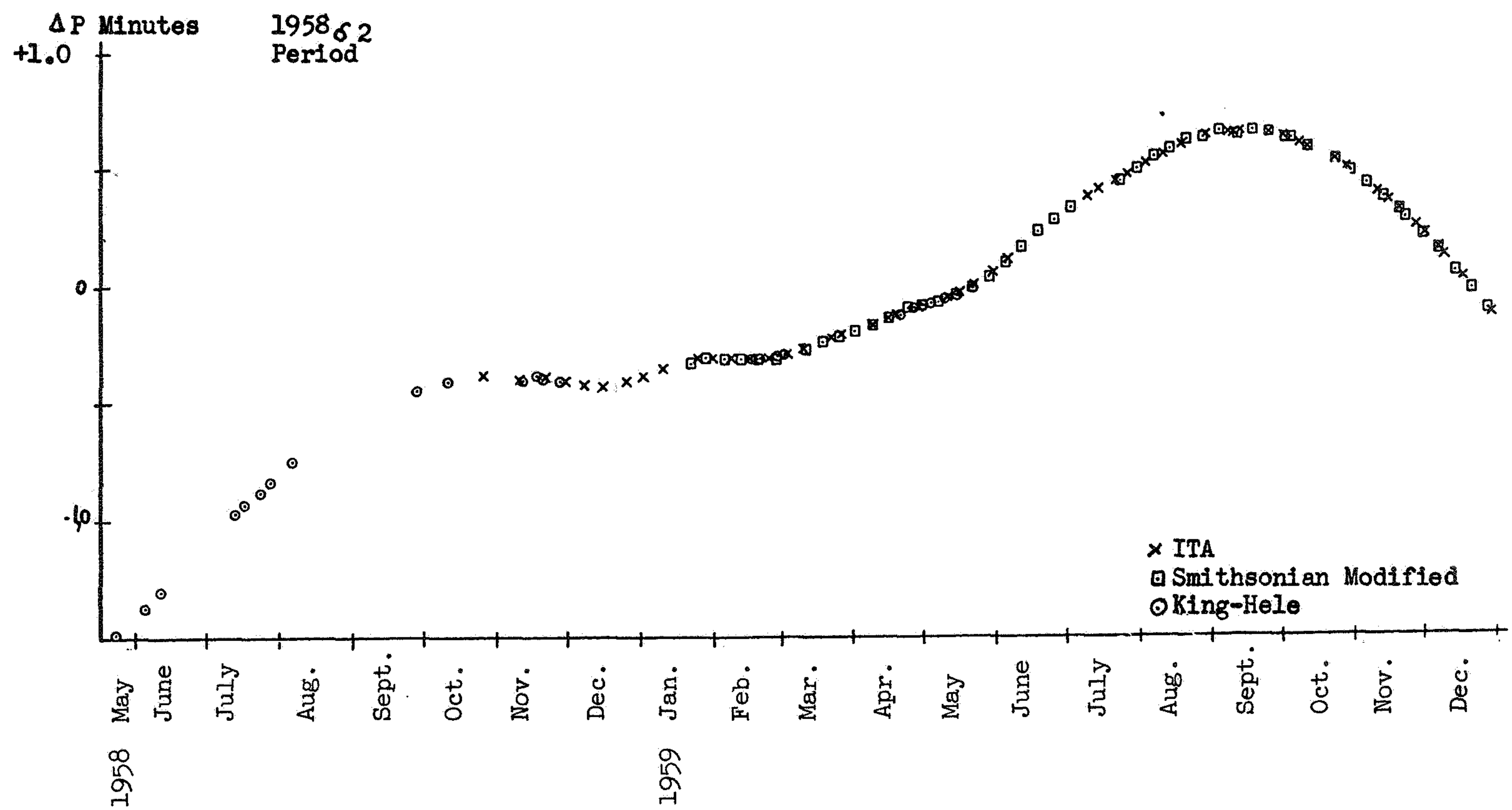

Figure 9.- (For the article by N. P. Slovokhotova.)

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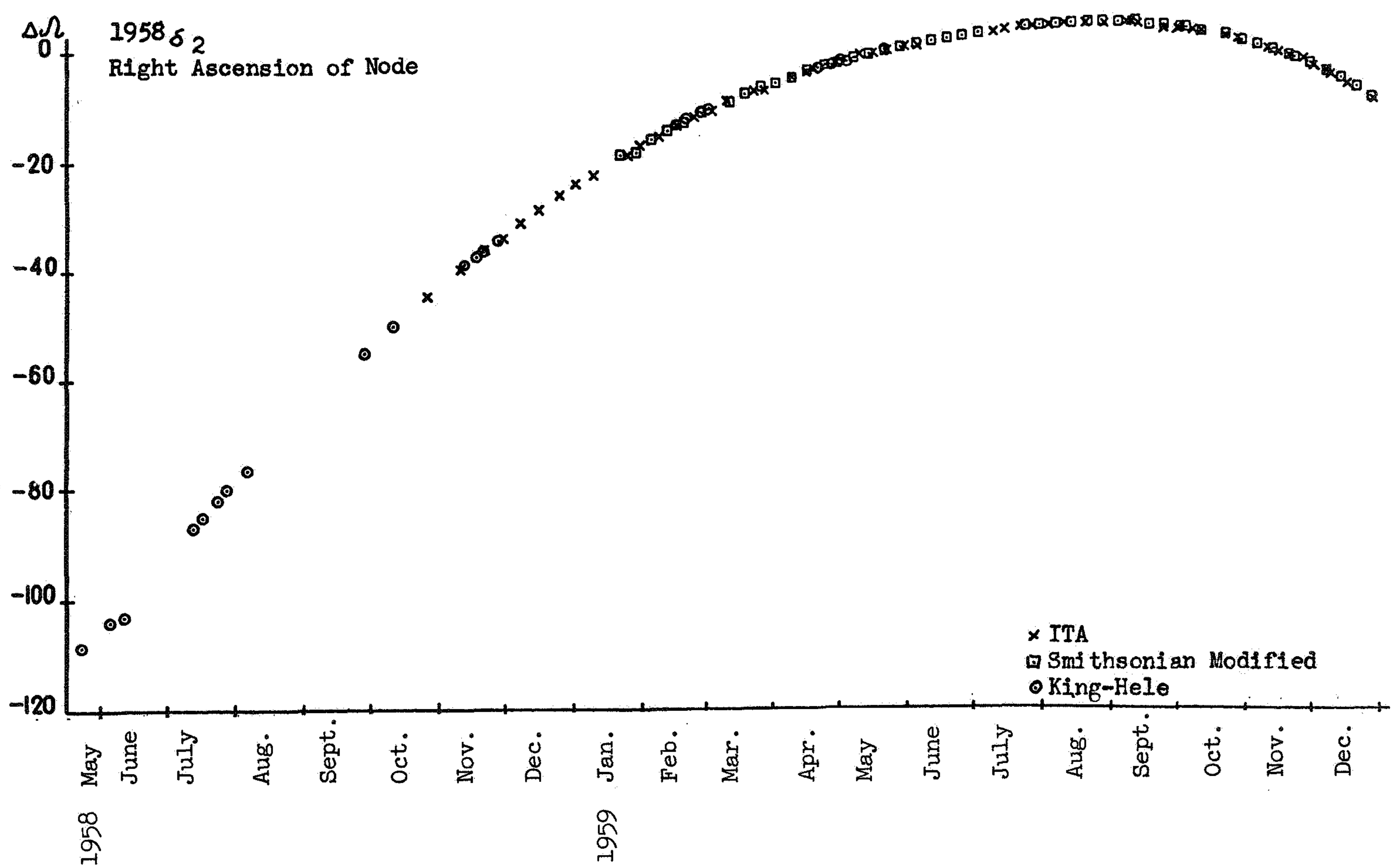

Figure 10.- (For the article by N. P. Slovokhotova.)

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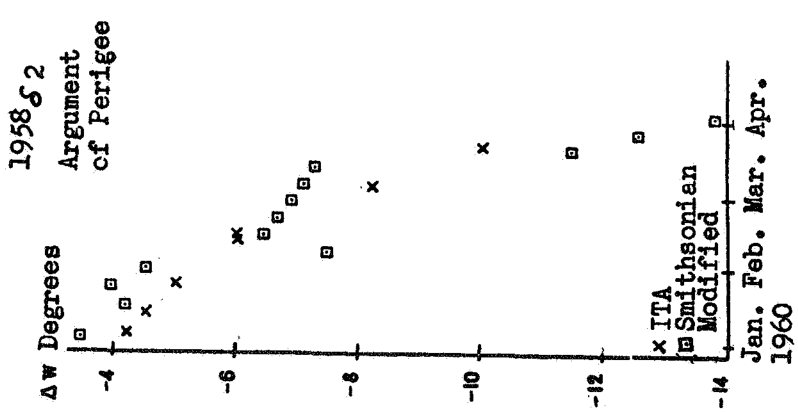

Figure 8(a).- (For the article by N. P. Slovokhotova.)

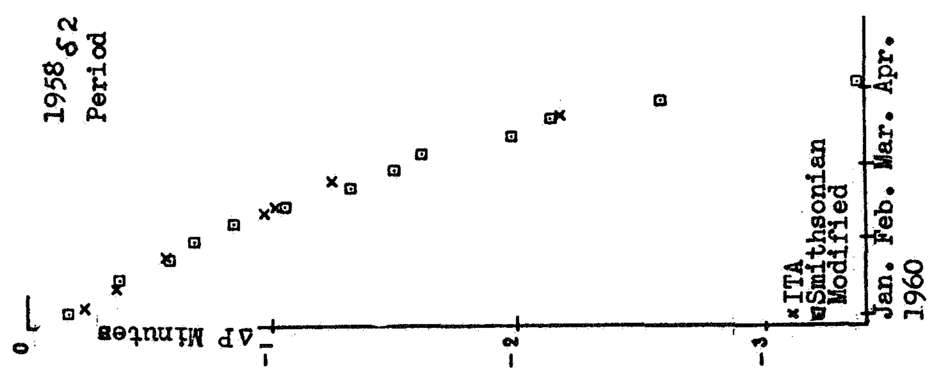

Figure 9(a).- (For the article by N. P. Slovokhotova.)

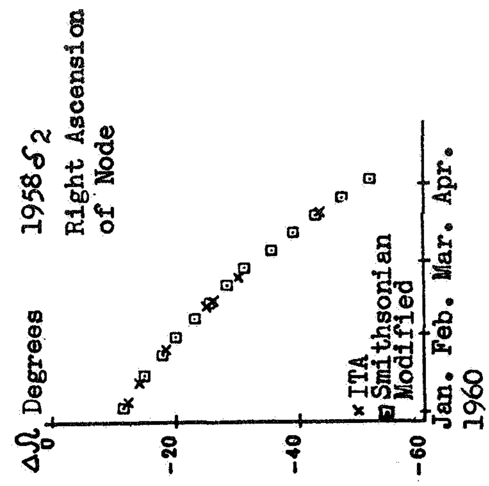

Figure 10(a).- (For the article by N. P. Slovokhotova.)

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